Self-consistent Mean Field Theory in Weakly Ionized Media

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ABSTRACT

We present a self-consistent mean field theory of the dynamo in 3D and turbulent diffusion in 2D in weakly ionized gas. We find that in 3D, the backreaction does not alter the beta effect while it suppresses the alpha effect when the strength of a mean magnetic field exceeds the critical value $B_c \sim \sqrt{\nu_{in}\tau_n\langle v^2\rangle/R_m}$. Here, ν_{in}, τ_n, and R_m are ion-neutral collision frequency, correlation time of neutrals, and magnetic Reynolds number, respectively. These results suggest that a mean field dynamo operates much more efficiently in weakly ionized gas where $\nu_{in}\tau_n \gg 1$, compared to the fully ionized gas. Furthermore, we show that in 2D, the turbulent diffusion is suppressed by back reaction when a mean magnetic field reaches the same critical strength B_c, with the upper bound on turbulent diffusion given by its kinematic value. Astrophysical implications are discussed.

Subject headings: ISM:magnetic fields—MHD—turbulence

1. Introduction

One of the most outstanding problems in the astrophysical MHD is to explain the origin of ubiquitous magnetic fields in stars, galaxies, interstellar medium (ISM), etc. These magnetic fields are often observed to be coherent on scales much larger than the characteristic scale of turbulence, with their energy being comparable with fluid kinetic energy (i.e., in equipartition). For instance, galactic magnetic fields are thought to have coherent magnetic fields with comparable strength to fluctuations. The major stumbling block to explaining these coherent magnetic fields by dynamo action in fully ionized gas is its tendency of generating too strong fluctuations, which unfortunately inhibit the growth of a coherent (mean) component by back reaction (Lorentz force) when the strength of a mean magnetic field is far below equipartition value – the so–called alpha quenching problem (Cattaneo & Hughes 1996; Gruzinov & Diamond 1994, 1996).

It is, however, largely unknown whether and/or how backreaction constrains alpha effect in weakly ionized medium, such as the galaxies, ISM, molecular clouds, etc, with ambipolar drift (slippage between magnetic fields and the bulk of fluid (neutrals)). This is partly because almost all previous works on the effect of ambipolar drift invoked strong coupling approximation (the drift between ions and neutrals is balanced by Lorentz force due to sufficiently frequent collisions between the two), which makes ambipolar drift mainly act as a nonlinear diffusion. Thus, the ambipolar drift has been primarily advocated as a means of enhancing the effective diffusion rate over Ohmic value (e.g., Mestel & Spitzer 1956; Zweibel 1988). It is also attributed to the fact that the important dynamic effect of fluctuations (turbulence and Lorentz force back reaction) has often been neglected (c.f., Boss 2000; Fatuzzo & Adams 2002). Interestingly, these two factors come in together as strong coupling approximation is likely to break down on small scales (i.e., for fluctuations) where Alfven frequency is larger than the ion-neutral collision frequency (Kim 1997). The purpose of this Letter is to present a first self-consistent mean-field theory of the dynamo in weakly ionized gas, by incorporating these important synergistic effects of turbulence and back-reaction, without invoking strong coupling approximation. We shall demonstrate that ambipolar drift reduces alpha quenching, even overcoming it in certain (but extreme) cases.

Before proceeding to a mean field theory of the dynamo, some insight into the effect of ambipolar drift can be gained by considering the problem of diffusion of a mean magnetic field in the two dimensions (2D). In the case of fully ionized gas, it is well known that the turbulent diffusion in 2D is severely reduced by back reaction (Cattaneo & Vainshtein 1991; Gruzinov & Diamond 1994). In weakly ionized gas, the turbulent diffusion is still reduced below its kinematic value while ambipolar drift can increase the critical strength of a mean magnetic field (above which the diffusion is reduced) by a factor of $\sqrt{\nu_{in}\tau_n}$ (Kim 1997). Here, ν_{in} and τ_n are the ion-neutral collision frequency and correlation time of neutrals. Thus, ambipolar drift offers the possibility of dissipating a mean magnetic field at turbulent rate for sufficiently large $\nu_{in}\tau_n$. In this Letter, we also provide a self-consistent mean field theory for the diffusion of a mean magnetic field in 2D in weakly ionized gas, which not only confirms the numerical results of Kim (1997) but also generalizes the perturbation analysis therein.

The remainder of the Letter is organized as follows. Section 2 contains the derivation of the effective dissipation rate of a mean magnetic field with ambipolar drift in 2D. We provide a mean field dynamo theory in weakly ionized gas in §3, by deriving the analytic expressions for alpha and beta effects. Concluding remarks are provided in §4.

2. TURBULENT DIFFUSION OF MEAN MAGNETIC FIELD IN 2D

In weakly ionized medium, Ohm's law is valid provided that the velocity of ions is used in the calculation of current. So, to consistently treat this problem, one needs to solve the momentum equation for ions and for neutrals (without Lorentz Force), together with induction equation for magnetic fields. Since $\nu_{in}/\rho_n = \nu_{ni}/\rho_i$ and $\rho_i/\rho_n \ll 1$ for weakly ionized medium, the neutral-ion collision frequency can be neglected. Here, ρ_n and ρ_i are the density of neutrals and ions, respectively, and ν_{in} and ν_{ni} are ion-neutral and neutral-ion collision frequency, respectively. Thus, the momentum equation for the neutrals is entirely decoupled from that of the ions as well as from the induction equation. We thus assume that neutral velocity is turbulent with a prescribed statistics, and then solve the momentum equation for ions, which evolves self-consistently by frictional coupling to neutrals and by Lorentz force. Note that we are not invoking the strong coupling approximation.

In 2D, we work with ion vorticity ω ($\omega\hat{z} = \nabla \times \mathbf{v}$) and magnetic potential A ($\mathbf{B} = \nabla \times (A\hat{z})$), which are governed by the following set of two equations (in dimensionless form):

$$
\begin{aligned}
(\partial_t + \mathbf{v}\cdot\nabla)\omega &= -(\mathbf{B}\cdot\nabla)\nabla^2 A + \chi\nabla^2\omega + \gamma(N-\omega)\,, \\
(\partial_t + \mathbf{v}\cdot\nabla)A &= \eta\nabla^2 A\,.
\end{aligned} \tag{1}
$$

Here, $\mathbf{v}$ is the ion velocity, N is the vorticity of neutrals, $\gamma = \nu_{in}\tau_n$ is the frictional coupling between ions and neutrals, η is Ohmic diffusivity, and χ is ion viscosity. We shall assume unity magnetic Prandtl number (i.e., $\eta = \chi$). By decomposing fields into large and small scale parts and assuming that there is no large scale displacement of the medium, we let $\mathbf{v} = \mathbf{v_0} + \mathbf{v} = \mathbf{v}, \omega = \omega_0 + \omega = \omega, N = N_0 + N = N, \mathbf{B} = \mathbf{B_0} + \mathbf{b}$, and $A = A_0 + a$. Here, subscript '0' denotes a mean component, averaged over the statistics of N. Using this decomposition, we can separate the system eq. (1) in large and small scale components to obtain

$$
\begin{aligned}
[\partial_t - \chi\nabla^2 + \gamma]\omega &= -(\mathbf{B_0}\cdot\nabla)\nabla^2 a + \gamma N\,, \\
[\partial_t - \eta\nabla^2]A_0 &= -\langle \mathbf{v}\cdot\nabla a\rangle = -\nabla\cdot\mathbf{G}\,, \\
[\partial_t - \eta\nabla^2]a &= -\mathbf{v}\cdot\nabla A_0\,.
\end{aligned} \tag{2}
$$

Here, angular brackets denote the average over the statistics of N; in the first equation, the term $\nabla^2 A_0$ was neglected due to the large-scale variation of A_0; in the last equation, the term $\mathbf{v}\cdot\nabla a - \langle\mathbf{v}\cdot\nabla a\rangle$ was dropped (quasi linear approximation). $\mathbf{G} = \langle\mathbf{v}a\rangle$ is the flux of magnetic potential, which determines the evolution (effective diffusion) of A_0. To obtain $\mathbf{G}$ in terms of mean quantities, we rewrite it as:

$$
\mathbf{G} = \langle\mathbf{v}\int dt\partial_t a\rangle + \langle\int dt\partial_t \mathbf{v}a\rangle = \mathbf{G}_1 + \mathbf{G}_2\,. \tag{3}
$$

Here, $\mathbf{G}_1$ is a kinematic part while $\mathbf{G}_2$ comes from the back reaction of the flow onto the magnetic potential. It is easy to check $\mathbf{G}_1 = -\frac{\tau}{2}\langle v^2\rangle\nabla A_0$, by using τ approximation

(Gruzinov & Diamond 1994, 1996), namely by replacing the time derivative by $\frac{1}{\tau}$. The expression for $\mathbf{G}_1$ is just the standard beta effect in 2D. It is interesting to express $\mathbf{G}_1$ in terms of N since the statistics of the latter can be prescribed. For simplicity, we assume the statistics of N to be stationary with a delta-function power spectrum around $k = k_0$ as follows:

$$\langle N(\mathbf{k_1}, t)N(\mathbf{k_2}, t)\rangle = \frac{\langle N^2\rangle}{2\pi k_0}\delta(\mathbf{k_1} + \mathbf{k_2})\delta(k_1 - k_0)\,. \tag{4}$$

By taking spatial Fourier transform of the first equation of eq. (2) without the Lorentz force term, and by using eq. (4), we obtain

$$\langle v^2\rangle = [\frac{\tau\gamma}{1+\tau\gamma}]^2\frac{\langle N^2\rangle}{k_0^2}\,. \tag{5}$$

Therefore, the effective diffusion coefficient in the kinematic limit is given by $\beta_0 = -\frac{\mathbf{G_1}}{\nabla A_0} = \frac{\tau}{2}[\frac{\tau\gamma}{1+\tau\gamma}]^2\frac{\langle N^2\rangle}{k_0^2}$. Note that β_0 takes its maximum value when neutrals and ions are strongly coupled with $\gamma\tau \gg 1$. This is a natural consequence of the assumption that ions obtain their kinetic energy through frictional coupling to neutrals. Thus, crudely put, β_0 is reduced by a factor $[\frac{1+\tau\gamma}{\tau\gamma}]^2$. If there were an independent energy source for ions, this would no longer be true.

To compute $\mathbf{G}_2$, we incorporate Lorentz force in the first equation of eq. (2) and take the Fourier transform to obtain:

$$\tilde{\omega}(\mathbf{k}) = \frac{\tau}{1+\tau\gamma}[-\varepsilon_{ij3}\int d\mathbf{k}'(\mathbf{k}-\mathbf{k}')_j\tilde{A}_0(\mathbf{k}-\mathbf{k}')k_i'k'^2\tilde{a}(\mathbf{k}') + \gamma\tilde{N}(\mathbf{k})]\,, \tag{6}$$

from which $\mathbf{G}_2$ follows as:

$$\begin{aligned}\mathbf{G}_2 &= -i\int d\mathbf{k}d\mathbf{k}'e^{[i(\mathbf{k}+\mathbf{k})\cdot\mathbf{x}]}\frac{k_l}{k^2}\varepsilon_{3lm}\langle\tilde{\omega}(\mathbf{k})\tilde{a}(\mathbf{k}')\rangle \qquad (7)\\ &= -\frac{\tau}{2(1+\tau\gamma)}\langle a\nabla^2 a\rangle\nabla A_0 - \frac{i\tau\gamma}{1+\tau\gamma}\int d\mathbf{k}d\mathbf{k}'e^{[i(\mathbf{k}+\mathbf{k})\cdot\mathbf{x}]}\frac{k_l}{k^2}\varepsilon_{3lm}\langle\tilde{N}(\mathbf{k})\tilde{a}(\mathbf{k}')\rangle\,.\end{aligned}$$

To calculate the first part of the RHS, we assumed that the magnetic potential fluctuations were isotropic and homogeneous. Since the neutrals are unlikely to be correlated with the magnetic field, the second part of the preceding equation can be neglected, leading to

$$\beta = \beta_0 + \frac{\tau}{2(1+\tau\gamma)}\langle a\nabla^2 a\rangle = \beta_0 - \frac{\tau}{2(1+\tau\gamma)}\langle b^2\rangle\,. \tag{8}$$

Compared to $\beta = \tau\langle v^2 - b^2\rangle/2$ in the fully ionized gas, the contribution from the backreaction in eq. (8) involves a multiplicative factor $1/(1+\tau\gamma)$. It is because the response of ions and magnetic field are different due to frictional coupling of ions to neutrals (see eq. (2)). To express $\langle a\nabla^2 a\rangle$ in terms of large scale quantities, we use Zeldovich theorem (Zeldovich 1957), which can be derived from the conservation of $\langle A^2\rangle$ in 2D-ideal MHD (by multiplying the third equation of eq. (2) by A and taking average over large scales) as

$$\eta\langle a\nabla^2 a\rangle = \langle a\mathbf{v}\rangle\cdot\nabla A_0 = -\beta(\nabla A_0)^2\,. \tag{9}$$

Thus, from eqs. (8) and (9), we obtain $\partial_t A_0 = (\eta+\beta)\nabla^2 A_0$ with

$$\beta \;=\; \frac{\beta_0}{1+\frac{\tau}{\eta(1+\gamma\tau)}(\nabla A_0)^2}\,. \tag{10}$$

Note that $(\eta+\beta)$ is the total effective diffusivity of A_0. In the case of weak coupling limit ($\tau\gamma \ll 1$), the previous equation reduces to the beta suppression in fully ionized gas for a given β_0. Note, however, that β_0 itself is proportional to $(\tau\gamma)^2/(1+\tau\gamma)^2 \sim (\tau\gamma)^2$. In the opposite strong coupling limit ($\tau\gamma \gg 1$), one recovers an expression similar to that of (Kim 1997) as $\beta \sim \beta_0/(1+B_0^2/\eta\gamma)$. Thus, back-reaction becomes insignificant when large scale magnetic field is weak enough so as to satisfy the following condition:

$$(\nabla A_0)^2 = B_0^2 \ll \eta\gamma = \frac{\gamma}{R_m}\langle v^2\rangle\,, \tag{11}$$

where $R_m = \tau\langle v^2\rangle/\eta$ is the magnetic Reynolds number. Thus, the critical strength of mean magnetic field for the suppression of β effect is $\gamma\langle v^2\rangle/R_m$, which is larger by a factor of γ than that in the fully ionized gas. Note that the turbulent diffusivity can reach its kinematic value $\beta_0 = \tau\langle v^2\rangle/2$ as $\gamma\tau_n \to \infty$ but can never be greater. That is, β_0 is the *upper limit* on β. These results agree with Kim (1997).

3. MEAN FIELD DYNAMO THEORY IN 3D

We now provide a mean field dynamo theory in weakly ionized gas in 3D, by self-consistently computing the alpha and beta effects. As previously, we use quasi-linear theory to obtain the following set of non-dimensionalized equations for fluctuations and mean field (denoted by a subscript '0'):

$$\begin{aligned}
[\partial_t + \gamma - \chi\nabla^2]\mathbf{v} &= \mathbf{B}_0 \cdot \nabla\mathbf{b} + \mathbf{b}\cdot\nabla\mathbf{B}_0 - \nabla p + \gamma\mathbf{N}\,, \qquad (12)\\
[\partial_t - \eta\nabla^2]\mathbf{b} &= \nabla\times(\mathbf{v}\times\mathbf{B}_0)\,,\\
[\partial_t - \eta\nabla^2]\mathbf{B}_0 &= \nabla\times\langle\mathbf{v}\times\mathbf{b}\rangle = \nabla\times\mathbf{E}\,.
\end{aligned}$$

Here, $\mathbf{N}$ is neutral velocity (not vorticity); $\mathbf{E} = \langle\mathbf{v}\times\mathbf{b}\rangle = \alpha\mathbf{B}_0 - \beta\nabla\times\mathbf{B}_0$ is the electromotive force, which contains α and β effects. To compute $\mathbf{E}$, we again consider two parts – the kinematic part $\mathbf{E}_0$ and the part coming from the back reaction of the magnetic field onto the fluids $\mathbf{E}_1$:

$$\begin{aligned}
\mathbf{E} &= \langle\mathbf{v}\times\int dt\partial_t\mathbf{v}\rangle + \langle\int dt\partial_t\mathbf{v}\times\mathbf{b}\rangle \qquad (13)\\
&= \alpha_0\mathbf{B}_0 - \beta_0\nabla\times\mathbf{B}_0 + \langle\int dt\partial_t\mathbf{v}\times\mathbf{b}\rangle = \mathbf{E}_0 + \mathbf{E}_1\,,
\end{aligned}$$

where $\alpha_0 = -\frac{\tau}{3}\langle\mathbf{v}\cdot\nabla\times\mathbf{v}\rangle$ and $\beta_0 = \frac{\tau}{3}\langle v^2\rangle$ are the kinematic values (c.F., Krause & Rädler 1980). These two coefficients can again be expressed in terms of $\mathbf{N}$ as:

$$\begin{aligned}
\alpha_0 &= -\frac{\tau}{3}[\frac{\gamma\tau}{1+\tau\gamma}]^2\langle\mathbf{N}\cdot\nabla\times\mathbf{N}\rangle\,, \qquad (14)\\
\beta_0 &= \frac{\tau}{3}[\frac{\gamma\tau}{1+\tau\gamma}]^2\langle N^2\rangle\,. \qquad (15)
\end{aligned}$$

The computation of $\mathbf{E}_1$ can most easily be done in Fourier space because of the pressure term. Thus, we write the equation for the velocity in Fourier space and then plug it into the electromotive force expression to obtain:

$$\begin{aligned}
E_{1\alpha} &= \frac{i\tau}{1+\tau\gamma}\,\epsilon_{\alpha\beta\gamma}\int d\mathbf{p}\int d\mathbf{q}\;\Gamma_{\beta\lambda\mu}(\mathbf{k}-\mathbf{p})B_{0\lambda}(\mathbf{q})\langle b_\mu(\mathbf{k}-\mathbf{p}-\mathbf{q})b_\gamma(\mathbf{p})\rangle\,, \qquad (16)\\
\Gamma_{\alpha\beta\gamma}(\mathbf{k}) &= \delta_{\alpha\beta}k_\gamma + \delta_{\alpha\gamma}k_\beta - 2k_\alpha k_\beta k_\gamma/k^2\,.
\end{aligned}$$

To compute $E_{1\alpha}$, we assume that the statistics of small scale magnetic fields is homogeneous and isotropic but not necessarily invariant under plane reflection, with the following correlation function:

$$\langle b_\alpha(\mathbf{k}) b_\beta(\mathbf{k}')\rangle = \delta(\mathbf{k}+\mathbf{k}')[\frac{M(k)}{4\pi k^2}(\delta_{\alpha\beta} - k_\alpha k_\beta/k^2) + i\frac{F(k)}{8\pi k^4}\epsilon_{\alpha\beta\gamma}k_\gamma] = \delta(\mathbf{k}+\mathbf{k}')\Phi_{\alpha\beta}(\mathbf{k})\,, \quad (17)$$

where $M(k)$ is the magnetic energy spectrum tensor and $F(k)$ is the magnetic helicity spectrum tensor. By using eq. (17) in (16) and by keeping terms up to k (stretching and diffusion term), we obtain:

$$E_{1\alpha} = \frac{i\tau}{1+\tau\gamma}\,\epsilon_{\alpha\beta\gamma}B_{0\lambda}(\mathbf{k})\int d\mathbf{p}\;\Phi_{\mu\gamma}(\mathbf{p})[(2\frac{p_\beta p_\lambda p_\mu}{p^2} - \delta_{\beta\lambda}p_\mu - \delta_{\beta\mu}p_\lambda) + (\delta_{\beta\lambda} - \frac{2p_\beta p_\lambda}{p^2})k_\mu]\,. \quad (18)$$

Since all integrals with odd numbers of p_i vanish, $E_{1\alpha}$ reduces to:

$$\begin{aligned} E_{1\alpha} = \frac{i\tau}{1+\tau\gamma}\,\epsilon_{\alpha\beta\gamma}B_{0\lambda}(\mathbf{k})[k_\mu\int d\mathbf{p}\;\frac{M(p)}{4\pi p2}(\delta_{\mu\gamma} - p_\mu p_\gamma/p^2)(\delta_{\beta\lambda} - \frac{2p_\beta p_\lambda}{p^2}) \qquad (19)\\ +i\int d\mathbf{p}\;\frac{F(p)}{8\pi p4}\epsilon_{\mu\gamma\delta}p_\delta(2\frac{p_\beta p_\lambda p_\mu}{p^2} - \delta_{\beta\lambda}p_\mu - \delta_{\beta\mu}p_\lambda)]\,. \end{aligned}$$

The first part (proportional to k_μ), contributing to β, vanishes when integrated over angles, while the second part gives the correction term to α due to back reaction. Thus, there is no change in β, namely, *the turbulent diffusion is not affected by the back reaction of small scale magnetic fields in 3D with ambipolar drift!* This result sharply contrasts to the claim made in the literature, based on strong coupling approximation, that ambipolar drift enhances the diffusion of a mean magnetic field in 3D (e.g., Subramanian 1998). Note that the result for fully ionized gas (Gruzinov & Diamond 1996) is recovered simply by taking the limit $\tau\gamma \to 0$, but by keeping β_0 constant. On the other hand, the surviving part of E_1, contributing to α, reads:

$$E_{1\alpha} = \frac{\tau}{1+\tau\gamma}\frac{\langle \mathbf{b}\cdot\nabla\times\mathbf{b}\rangle}{3}B_{0\alpha}\,. \quad (20)$$

Therefore, α effect, including the back reaction of the magnetic field, follows from eqs. (13), (14), and (20) as

$$\alpha = \alpha_0 + \frac{\tau}{1+\tau\gamma}\frac{\langle \mathbf{b}\cdot\nabla\times\mathbf{b}\rangle}{3}\,. \quad (21)$$

Note that only the helical (resp. non-helical) part of the magnetic spectrum is involved in the alpha (resp. beta) effect since α (resp. β) is a pseudo-scalar (resp. scalar). Compared to the fully ionized gas, the contribution from the current helicity to α contains the additional multiplicative factor of $1/(1+\tau\gamma)$. This is again because the response time of ions is different from that of magnetic fields due to frictional coupling to neutrals (see eq. (12)). Thus, it is very likely that the cancellation between fluid and current helicity for Alfven waves (as happens in fully ionized case with $\gamma = 0$) may be avoided for $\gamma\tau > 1$, thereby reducing the suppression of α effect. This shall be shown below. To close the expression for α, we need to express the current helicity in terms of mean magnetic fields. To do so, we use the topological invariant of mean magnetic helicity $\langle \mathbf{a} \cdot \mathbf{b} \rangle$ in 3D, from which an analog of Zeldovich theorem can be derived as

$$\eta \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle = -\langle \mathbf{v} \times \mathbf{b} \rangle \cdot \mathbf{B_0} = -\alpha B_0^2 + \beta_0 \mathbf{B_0} \cdot \nabla \times \mathbf{B_0} \,. \tag{22}$$

Finally, combining eqs. (21) and (22), we obtain the non-linear α-effect expression for 3D-MHD with ambipolar drift:

$$\alpha = \frac{\alpha_0 + \frac{\tau\beta_0}{3\eta(1+\tau\gamma)} \mathbf{B_0} \cdot \nabla \times \mathbf{B_0}}{1 + \frac{\tau}{3(1+\tau\gamma)} \frac{B_0^2}{\eta}} \,. \tag{23}$$

The previous equation recovers α in the case of fully ionized gas as $\gamma\tau \to 0$. In the strong coupling limit ($\gamma\tau \gg 1$), α effect is suppressed when eq. (11) is satisfied. Therefore, the critical strength of mean magnetic field for the suppression of α effect is $\gamma\langle v^2 \rangle / R_m$, larger by a factor of γ, compared to the case of fully ionized gas. This has significant implications for a mean field dynamo in the galaxy, ISM, etc where the bulk of fluid consists of neutrals with $\gamma \gg 1$ (see §4 for more discussion).

4. CONCLUSION

We have presented self-consistent mean field theory of the turbulent diffusion (in 2D) and the dynamo (in 3D) in weakly ionized gas, by incorporating turbulence and back reaction of fluctuating magnetic fields. The key results are that in 3D, the backreaction does not alter the beta effect while it suppresses the alpha effect when the strength of a mean magnetic field exceeds the critical value $B_c^2 \sim \gamma\langle v^2\rangle/R_m$. This critical value is larger than that $B_c^2 \sim \langle v^2\rangle/R_m$ in the case of the fully ionized gas for $\gamma = \nu_{in}\tau_n > 1$. Alternatively put, the suppression factor for the alpha effect is reduced by a factor of γ, compared to the fully ionized gas. The upper bound on α is given by its kinematic value $\alpha_0 = -\tau\langle \mathbf{v}\cdot(\nabla\times\mathbf{v})\rangle/3$. In 2D, the turbulent diffusion (β effect) was shown to be suppressed by back reaction when a mean magnetic field reaches the same critical value $B_c^2 \sim \gamma\langle v^2\rangle/R_m$, with the upper bound on turbulent diffusion given by the kinematic value $\beta_0 = \tau\langle v^2\rangle/2$. These results are consistent with those in Kim (1997).

Therefore, in weakly ionized gas, the degree of alpha quenching (in 3D) and the suppression of turbulent diffusion (in 2D) crucially depends on $\gamma = \nu_{in}\tau_n$ in addition to R_m, i.e., the property of medium such as ionization, turbulence, etc. As $\nu_{in} \sim 10^{-2} n_n$ cm^3/yr (e.g., see Kim 1997), $\nu_{in}\tau_n \sim 10^5 n_n$ for $\tau_n \sim 10^7$yr. Here, n_n is the number density of neutrals in unit of cm^{-3}. Therefore, in the limit of a very low ionization, the alpha quenching (and beta quenching in 2D) can be significantly reduced. For instance, in the case of young galaxy with $n_n \sim 1\mathrm{cm}^{-3}$, $L \sim 100$pc, $v \sim 10$km/s, and $T \sim 10^4$K, $\eta = 10^7(T/10^4)^{-3/2}\mathrm{cm}^2/\mathrm{s}$ $\sim 10^7\mathrm{cm}^2/\mathrm{s}$, and $R_m \sim vL/\eta \sim 10^{19}$. Thus, $\nu_{in}\tau_n/R_m \sim 10^5/R_m \sim 10^{-14}$, with the critical strength of mean field $B_c \sim 10^{-7} \times \sqrt{\langle v^2\rangle}$, which is too weak. However, for dark molecular clouds with $n_n \sim 10^7\mathrm{cm}^{-3}$, $L \sim 1$pc, $v \sim 1$km/s, and $T \sim 10$K, $\eta = 3\times 10^{11}\mathrm{cm}^2/\mathrm{s}$ and $R_m \sim vL/\eta \sim 10^{12}$. Thus, $\nu_{in}\tau_n/R_m \sim 1$, giving $B_c \sim \sqrt{\langle v^2\rangle}$! Therefore, in this extremely low ionized gas, a mean field dynamo may work efficiently without alpha quenching.

These results essentially come from the fact that the turbulence in weakly ionized gas does not become Alfvenic as the motion of ions undergoes frictional damping due to the coupling to neutrals (or, due to ambipolar drift). This is quite similar to what happens in a very viscous fluid with $R_e \ll R_m$ (R_e is the Reynolds number), in which case the suppression factor for alpha quenching is also reduced because of viscous damping of ion velocity (Kim 1999). Since $R_e \ll R_m$ in the galaxies with a low ionization, the combined effect of ambipolar drift and viscous damping of fluid may render the mean dynamo sufficiently efficient, without severe alpha quenching. This interesting problem will be investigated in a future paper.

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